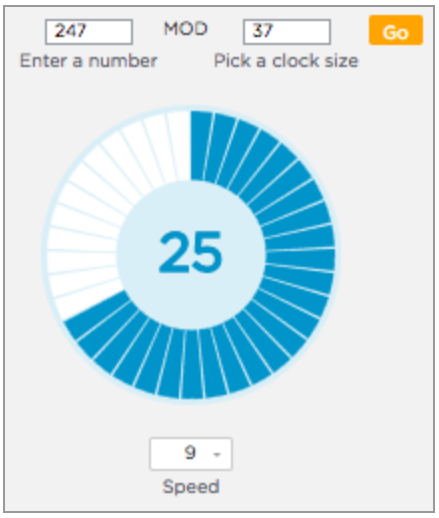
Name(s)_____ Period _____ Date _____

Activity Guide - Multiplication + Modulo

Summary

In this activity you'll you'll multiply numbers as input into the modulo operation and explore some interesting properties that relate to cryptography.



Goal: Understand how multiplication + modulo can be used to make computationally-hard-to-crack encryption.

Tools:

- **Calculator:** you probably want a calculator handy for multiplying big numbers
- **The "Mod Clock" widget** in code studio (pictured at right)

Assumption: You have been introduced to the modulo operation and the "clock" analogy for it.

Step 1: Experiment with the Mod Clock

Goal: familiarize yourself with properties of the Modulo operation

Get your feet wet - play

- Try inputting different values into the mod clock for *both* the "number" and the "clock size".
- Try big numbers and small numbers for both

Questions:

1. Using a clock size of 50, write a list of 5 numbers that produce a result of 0.

2. With clock size of 50 how many total numbers are there that produce a result of 0? (If the list is short, write it out. If the list is long, describe a pattern of what the numbers are).

3. Using a clock size of 13, can you find a number to input that produces a *result* of 13? (If so, what is it? If not, why not?)

4. Using a clock size of 13, find the answers to the following:

1 MOD 13	
10 MOD 13	
100 MOD 13	

1,000 MOD 13	
10,000 MOD 13	
100,000 MOD 13	

Are these results surprising or interesting? Why or why not?

Step 2: Toward encryption - Use multiplication to produce inputs

Experiment - Small changes to inputs, big changes to outputs.
Using a clock size of 37, let's multiply two numbers (we'll call them A and B) to use as input, then make small changes to each while holding the other constant. We'll always use the formula **`A * B MOD M.`** We'll start with A=20 and B=50 and M=37. So here is the first result...

`20 * 50` MOD 37 = **`1`**

Now find in the rest of these values making small adjustments to A and B individually.
*Use a calculator, if necessary, to compute A * B. Use the Mod Clock to compute the modulus of the result.*

Increment A	
21*50 MOD 37	
22*50 MOD 37	
23*50 MOD 37	

Increment B	
20***51** MOD 37	
20***52** MOD 37	
20***53** MOD 37	

Result: What do you notice about these results? Is there a pattern? Could you predict the result of **`25 * 50 MOD 37`**?

Experiment 2 - Guessing inputs is hard?: Using a clock size of 101, we'll give you the value of A and even hold the result of the modulo operation constant. **Your task:** find a value for B (the blank) that makes the math work out.

1) **`2 * ____ MOD 101 = 1`**

2) **`3 * ____ MOD 101 = 1`**

3) **`4 * ____ MOD 101 = 1`**

Takeaways:
Solving modulus equations like **`2 * ___ MOD 101 = 1`** is "hard" because you can't solve it like a typical equation. There are no easy patterns or shortcuts like other equations you might see in a math class. As you learned in step 1 (hopefully) there is an infinite list of single values for which **`___ MOD 101 = 1`**. (The list is 1, 102, 203, 304, 405...etc). With multiplication, to solve **`2 * ___ MOD 101 = 1`** you end up randomly guessing to find some number to multiply by 2 that gives you a result in that list.

Things get especially "hard" when you use a prime number as the clock size. Thanks to some special properties of prime numbers **with a prime clock size there's only one solution to each modulus equation.** You are guaranteed that there is the number less than the clock size itself, but there are still 100 different values you have to try. With a **brute force search** you could go through them all in a couple minutes. But what if the clock size were a 50-digit prime number?

Encryption!
Whenever you have a problem for which the only way to solve it is by random guessing or brute force search over a large range of values, you have a candidate for a encryption. Next you'll get to try it!